1. Convex functions and Jensen's inequality

A function $f\colon \mathbb{R}^n \to \mathbb{R}$ is *convex* if for any $x, y \in \mathbb{R}^n$ and for any $\lambda \in [0,1]$ it holds

$$\lambda f(x) + (1-\lambda) f(y) \geq f(\lambda x + (1-\lambda) y).$$

A convex function f is known to satisfy the following inequality: For any integer $k \geq 1$, any $x_1, x_2, \dots, x_k \in \mathbb{R}^n$, and any $\lambda_1, \lambda_2, \dots, \lambda_k \in \mathbb{R}$ with $\sum_{i=1}^{k} \lambda_i = 1$, it holds that

$$\sum_{i=1}^{k} \lambda_i f(x_i) \geq f\left(\sum_{i=1}^{k} \lambda_i x_i\right).$$

This inequality is called *Jensen's inequality*.

2. Big-O notation

Let $f, g\colon \mathbb{N} \to \mathbb{R}$ be two functions. We write $f(n) = \mathrm{O}(g(n))$ if f is bounded by g from above in the order of magnitude. Formally speaking, we say $f(n) = \mathrm{O}(g(n))$ if there exist k and M (which depend on f, g) such that for all $n > k$ it holds that $|f(n)/g(n)| \leq M$. If $g(n) = \mathrm{O}(f(n))$, we write $f(n) = \Omega(g(n))$. If $f(n) = \mathrm{O}(g(n))$ and $f(n) = \Omega(g(n))$, then we write $f(n) = \Theta(g(n))$.

Another frequently encountered notation is the little-o. We write $f(n) = \mathrm{o}(g(n))$ if f is negligible compared to g in the order of magnitude (or f is less than g in the order of magnitude). Formally speaking, we say $f(n) = \mathrm{o}(g(n))$ if $\lim_{n\to\infty} |f(n)/g(n)| = 0$.

Here is the summary.

Notation	Definition	Interpretation
$f(n) = \mathrm{O}(g(n))$	$\exists k, M\ \forall n > k$: $\lvert f(n)/g(n)\rvert \leq M$	f is at most g in the order of magnitude.
$f(n) = \Omega(g(n))$	$\exists k, M\ \forall n > k$: $\lvert g(n)/f(n)\rvert \leq M$	f is at least g in the order of magnitude.
$f(n) = \Theta(g(n))$	$f(n) = \mathrm{O}(g(n))$ and $f(n) = \Omega(g(n))$	f has the same order of magnitude as g.
$f(n) = \mathrm{o}(g(n))$	$\lim_{n\to\infty} \lvert f(n)/g(n)\rvert = 0$	f is less than g in the order of magnitude.

Another (yet interesting) remark is that O and o in these notations are not "Ohs" in the Latin alphabet. They are "Omicrons" in the Greek alphabet.