FINITE CO-DIMENSIONAL BANACH SPACES AND SOME BOUNDED RECOVERY PROBLEMS

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In this paper we study the projections and some recovery problem of a finite CO-dimensional Banach spaces in terms of the projection of their complementations, more precisely we study the following problems:

(1) If $Y$ is a finite Co-dimensional subspace of a Banach space $X$ and $Z$ is its complementation, is for every projection $P_0$ from $X$ onto $Z$ and every $\epsilon > 0$ there a projection $P$ from $X$ onto $Y$ satisfying

$$\|P\| \leq 1 + (1 + \epsilon)\|P_0\|?$$

(2) If $X$ is a Banach space, $x \in X$, $Y$ is an n-CO-dimensional subspace of $X$ and $\{(f_i, x_i)\}_{i=1}^{n}$ is the Auerbach system of the complementation $Z$ of $Y$ in $X$, is there an element $y \in Y$ satisfying the following two conditions

(i) $\hat{f}_i(y) = \hat{f}_i(x) \forall i \in \{1, 2, \ldots, n\}$, where $\hat{f}_i$ is the Hahn-Banach extension of $f_i$ on $X$,

(ii) $\|y\| \leq M\|x\|$ for some constant $M$?

And we study the restrictions placed on the constant $M$ as a function of $X$ and $Y$.

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